Small-Angle Photoproduction and Conspiracy*

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In any reaction of the class $\pi+B \to V+B$ or $V+B \to \pi+B$, nonconspiring amplitudes with a meson helicity change ($h_V-h_\pi \neq 0$) exhibit a dip at small angles. Normally, this would simply mean that the zero-helicity-change amplitude dominates forward production. But in photoproduction, since the photon has no longitudinal component, there is no zero-helicity-change amplitude, and in the absence of conspiracy a forward dip will appear in the differential cross section. When conspiracy is present, the dip does not appear. Thus, forward photoproduction provides a good test for conspiracy.

I. INTRODUCTION

ONE way of looking at conspiracy in nucleon-nucleon scattering[1–4] is that without it, the "flip-flip" amplitude would vanish at $t=0$. Of course, the "flip-nonflip" amplitude also vanishes at $t=0$ on account of angular momentum conservation, so only the "nonflip-nonflip" amplitude would contribute. What conspiracy does is to allow the flip-flip amplitude to take on nonzero values at $t=0$, as permitted by angular-momentum conservation. Unfortunately, however, it has proved rather difficult to disentangle the experimental nonflip-nonflip and flip-flip effects and establish that conspiracy really occurs in nucleon-nucleon scattering.

Recently, one of us[5] has studied the class of reactions $\pi+N \to M+B$ where M is a 1^- or 2^+ meson and B is a baryon, and shown that in the absence of conspiracy, all amplitudes for nonzero helicity change $\mu=h_M-h_\pi$ have dips in the forward direction (i.e., they do not grow with the leading Regge power). Conspiracy removes the dips for those forward amplitudes which conserve angular momentum. While the technical details of this effect depend upon unequal-mass kinematics, the result is evidently similar to the nucleon-nucleon case—in the absence of conspiracy, nonzero helicity changes are suppressed. The application of the effect made by Jones[5] was to note that since exchange of natural parity trajectories [ρ, A_2, etc., with $P=(-1)^J$] does not contribute to the $\mu=0$ amplitude, exchange of these "top-ranking" trajectories will be suppressed at small angles if they do not conspire. The evidence that this does indeed happen[5] supports the belief that the nonets of vector and tensor trajectories do not conspire, but whether some *other* trajectories may be conspiring is again left obscure.

In the present paper we consider $\gamma N \to \pi N$. Since this is a special case of the class of reactions $\pi N \to VN$, time-reversed, the forward dip discovered by Jones should again apply to all amplitudes with $\mu \neq 0$ if there is no conspiracy. But photoproduction has the special feature that there is *no* amplitude with $\mu=h_\pi-h_\gamma=0$; therefore, the dip should show up directly in the cross section and does not have to be disentangled from a dominant nonflip contribution in the usual fashion.

Because of this special feature, photoproduction is an especially favorable reaction for establishing whether conspiracy exists. At present, there are three strong indications that conspiracy does exist in photoproduction:

(i) Halpern[6] has noted that the invariant amplitude conventionally labeled[7] A_1 vanishes at $t=0$ unless there is conspiracy. But the data, as discussed by Adler and Gilman[8] and by Halpern,[6] indicate that A_1 does *not* vanish at $t=0$.

(ii) Halpern has also derived a sum rule for the pion-nucleon coupling f_r,

$$f_r=\frac{2}{\pi}\int_{\nu_0}^{\infty} d\nu' \,\mathrm{Im}A_1^{(-)}(\nu', t=0)\,, \tag{1.1}$$

which evidently requires that A_1 not vanish at $t=0$.

(iii) Finally, there is our point that the differential cross section would exhibit a forward dip were there no conspiracy.[9] Actually, the differential cross section for $\gamma+p\to\pi^++n$ (Fig. 1) appears to be rising as one approaches small angles,[10] strongly suggesting conspiracy. As we discuss in Sec. III, measurements of the

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[7] J. S. Ball, Phys. Rev. **124**, 2014 (1961); **124**, 2014 (1961). The amplitudes A, listed in Table II, differ from those of Ball by a constant factor with the dimensions of nucleon mass.

[8] S. L. Adler and F. J. Gilman, Phys. Rev. **152**, 1460 (1966).

[9] This point has also been realized by S. Drell and J. Sullivan, Phys. Rev. Letters **19**, 268 (1967); and by B. Diu and M. Le Bellac, Kinematical Constraints on Regge Pole Residues, Orsay Report Th(198), 1967 (unpublished).

[10] M. Beneventano *et al.*, Nuovo Cimento **28**, 1464 (1963); S. D. Ecklund and R. L. Walker, Phys. Rev. **159**, 1195 (1967); G. Buschhorn *et al.*, Phys. Rev. Letters **18**, 571 (1967).

TABLE I. Kinematic singularities and partial-wave expansions of the helicity amplitudes for photoproduction.[a]

Amplitude	$\|\lambda\|$	$\|\mu\|$	$\bar{K}(t)$	Partial-wave expansion	Dominant parity
$\bar{f}_{01,\frac{1}{2}\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}-\frac{1}{2}}{}^t$	0	1	$(t-\mu^2)$	$\sum(2J+1)b_J{}^+P_{J-1}{}^{(1,1)}(\cos\theta_t)$	$(-1)^J$
$\bar{f}_{01,\frac{1}{2}\frac{1}{2}}{}^t-\bar{f}_{01,-\frac{1}{2}-\frac{1}{2}}{}^t$	0	1	$(t-4M^2)^{1/2}/t^{1/2}*$	$\sum(2J+1)b_J{}^-P_{J-1}{}^{(1,1)}(\cos\theta_t)$	$(-1)^{J+1}$
$\bar{f}_{01,\frac{1}{2}-\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}\frac{1}{2}}{}^t$	1	1	$(t-\mu^2)/t^{1/2}*$	$\sum(J+\frac{1}{2})[a_J{}^-(P_{J-1}{}^{(2,0)}-P_{J-1}{}^{(0,2)})+a_J{}^+(P_{J-1}{}^{(2,0)}+P_{J-1}{}^{(0,2)})]$	$(-1)^J$
$\bar{f}_{01,\frac{1}{2}-\frac{1}{2}}{}^t-\bar{f}_{01,-\frac{1}{2}\frac{1}{2}}{}^t$	1	1	$(t-4M^2)^{1/2}$	$\sum(J+\frac{1}{2})[a_J{}^+(P_{J-1}{}^{(2,0)}-P_{J-1}{}^{(0,2)})+a_J{}^-(P_{J-1}{}^{(2,0)}+P_{J-1}{}^{(0,2)})]$	$(-1)^{J+1}$

[a] In the absence of conspiracy, the amplitudes marked by an asterisk should be multiplied by an extra factor t.

0° cross section at high energy should settle the question definitively.

In Sec. II we discuss the helicity amplitudes for photoproduction in the presence and absence of conspiracy. In Sec. III we show that the differential cross section has a forward dip unless conspiracy is present. In two appendices we discuss complications due to the zero mass of the photon, and an alternative method of deriving conspiracy conditions by using only crossing [rather than the usual methods based on invariant amplitudes or on $O(4)$].

II. PHOTOPRODUCTION AMPLITUDES WITH AND WITHOUT CONSPIRACY

In Reggeizing the helicity amplitudes for photoproduction, we follow the method initiated by Gell-Mann *et al.*, and developed by Hara and Wang.[11] The

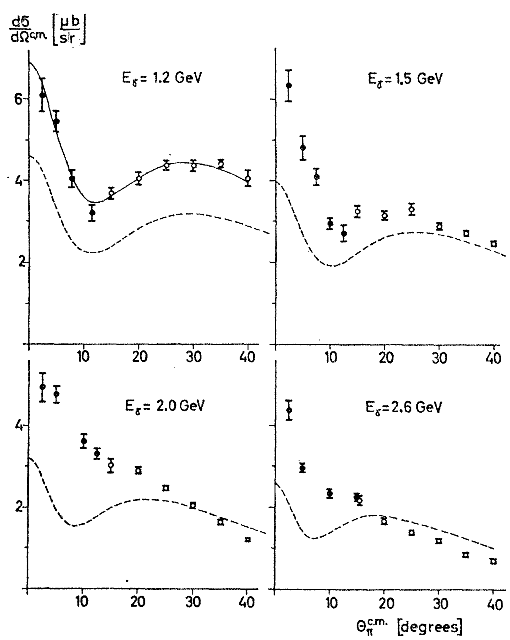

FIG. 1. Center-of-mass differential cross sections $(d\sigma/d\Omega)_{\gamma p\rightarrow\pi^+n}$ as a function of pion c.m. angle $\theta_\pi{}^{\mathrm{c.m.}}$ for different incident photon energies E_γ, reproduced from Buschhorn *et al.* (Ref. 10). The broken lines may be disregarded.

t-channel helicity amplitudes are written

$$f_{cd,ab}{}^t=(\sin\tfrac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\tfrac{1}{2}\theta_t)^{|\lambda+\mu|}\bar{f}_{cd,ab}{}^t\,, \qquad (2.1)$$

where $\lambda=a-b$ and $\mu=c-d$. The "parity-conserving" helicity amplitudes are written

$$\bar{f}_{cd,ab}{}^t\pm\bar{f}_{cd,-a-b}{}^t=\bar{K}_{cd,ab}{}^\pm(t)\tilde{f}_{cd,ab}{}^t(s,t)\,, \qquad (2.2)$$

where $\bar{K}$ is a kinematic factor given by Wang[12] and $\tilde{f}^t$ is the kinematic singularity-free amplitude which one actually Reggeizes.

The four independent combinations of helicity amplitudes needed to describe photoproduction, and the kinematic factor $\bar{K}$ for each of them, are listed in Table I. In evaluating $\bar{K}$, there are special problems associated with the zero mass of the photon, and the $\bar{K}$ listed in Table I is the limit as $m_V\rightarrow 0$ of $\bar{K}$ evaluated for the amplitudes with $m_V\neq 0$. The reasons why this choice has been made are given in Appendix A.

It is also straightforward to express the four invariant amplitudes A_i of Ball[7] in terms of the t-channel helicity amplitudes $\bar{f}^t$. This is done in Table II.

Now as $t\rightarrow 0$, let each helicity amplitude behave like the corresponding kinematic factor $\bar{K}(t)$ listed in Table I. The effect of this behavior on the A_i is easily deduced from Table II. One finds that all A_i can be nonzero at $t=0$. A_2 will have a $1/t$ singularity, however, unless the coefficient of the $1/t$ term satisfies the condition

$$\bar{f}_{01,\frac{1}{2}\frac{1}{2}}{}^t-\bar{f}_{01,-\frac{1}{2}-\frac{1}{2}}{}^t=-i(\bar{f}_{01,\frac{1}{2}-\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}\frac{1}{2}}{}^t)\,. \qquad (2.3)$$

This condition can be satisfied in either of two ways:

(i) By conspiracy among the different types of Regge trajectory involved in (2.3). This has been studied by Halpern,[6] Mitter,[13] and Sawyer.[14] Mitter and Sawyer find by an $O(4)$ analysis that the conspiracy must be of "Class III" in the classification of Freedman and Wang.[4]

(ii) By individual vanishing ($\sim t$) of the Regge residues on the left- and right-hand sides of (2.3). The extra factors of t in this "no-conspiracy" solution are indicated by the asterisk in Table I. Note that in this case, with $(\bar{f}_{01,\frac{1}{2}\frac{1}{2}}{}^t-\bar{f}_{01,-\frac{1}{2}-\frac{1}{2}}{}^t)$ and $(\bar{f}_{01,\frac{1}{2}-\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}\frac{1}{2}}{}^t)$

[11] M. Gell-Mann, M. Goldberger, F. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964); Y. Hara, *ibid.* **136**, B507 (1964); L. L. Wang, *ibid.* **142**, 1187 (1966).

[12] Note the change of convention here from Wang (Ref. 11) and Jones, (Ref. 5) who work with $\bar{f}_{cd,ab}{}^t\pm\bar{f}_{-c-d,ab}{}^t$. The amplitudes $\bar{f}_{-c-d,ab}{}^t$ and $\bar{f}_{cd,-a-b}{}^t$ are easily related by the use of Wang's Eq. (III5).

[13] P. K. Mitter, Phys. Rev. **162**, 1624 (1967).

[14] R. F. Sawyer, Phys. Rev. Letters **19**, 137 (1967).

TABLE II. Relation between invariant amplitudes A_i and t-channel helicity amplitudes.

$$A_1=\frac{2\sqrt{2}M}{(4M^2-t)(t-\mu^2)}\left[t(\bar{f}_{01,\frac{1}{2}\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}-\frac{1}{2}}{}^t)-2Mt^{1/2}(\bar{f}_{01,\frac{1}{2}-\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}\frac{1}{2}}{}^t)\right]$$

$$A_2=\frac{2\sqrt{2}M}{(t-4M^2)(t-\mu^2)}\left[(\bar{f}_{01,\frac{1}{2}\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}-\frac{1}{2}}{}^t)+\left(\frac{t-4M^2}{t}\right)^{1/2}(\bar{f}_{01,\frac{1}{2}\frac{1}{2}}{}^t-\bar{f}_{01,-\frac{1}{2}-\frac{1}{2}}{}^t)-\frac{2M}{t^{1/2}}(\bar{f}_{01,\frac{1}{2}-\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}\frac{1}{2}}{}^t)\right]$$

$$A_3=\frac{\sqrt{2}M}{(t-4M^2)^{1/2}(t-\mu^2)}\left[\bar{f}_{01,\frac{1}{2}-\frac{1}{2}}{}^t-\bar{f}_{01,-\frac{1}{2}\frac{1}{2}}{}^t\right]$$

$$A_4=\frac{2\sqrt{2}M}{(4M^2-t)(t-\mu^2)}\left[2M(\bar{f}_{01,\frac{1}{2}\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}-\frac{1}{2}}{}^t)-t^{1/2}(\bar{f}_{01,\frac{1}{2}-\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}\frac{1}{2}}{}^t)\right]$$

behaving like $t^{1/2}$ instead of $t^{-1/2}$, the amplitude A_1 in Table II vanishes at $t=0$. As stated in the Introduction, this is the basis of Halpern's arguments for the existence of conspiracy in photoproduction.

The equations we have written thus far have referred to helicity amplitudes, and that is all one needs to establish the properties of forward photoproduction in Sec. III. To express the conspiracy relations in terms of Regge trajectories, however, which is needed for Appendix A, and on emust make a partial wave expansion of the helicity amplitudes. The amplitudes $\bar{f}^t$ have the partial-wave expansion

$$\bar{f}_{cd,ab}{}^t=\sum_J(2J+1)T_{cd,ab}{}^J(t)P_{J-M}{}^{(|\lambda-\mu|,|\lambda+\mu|)}(\cos\theta_t)\,, \tag{2.4}$$

where

$$M=\text{maximum of }(|\lambda|,|\mu|)\,,$$

and P_{J-M} is the Jacobi polynomial. Partial-wave amplitudes with definite parity are

$$a_J{}^{\pm}\equiv T_{01,\frac{1}{2}-\frac{1}{2}}{}^J\pm T_{01,-\frac{1}{2}\frac{1}{2}}{}^J\,, \tag{2.5}$$

$$b_J{}^{\pm}\equiv T_{01,\frac{1}{2}\frac{1}{2}}{}^J\pm T_{01,-\frac{1}{2}-\frac{1}{2}}{}^J\,. \tag{2.6}$$

Here, amplitudes with $+$ superscripts have $P=(-1)^J$, and amplitudes with $-$ superscripts have $P=(-1)^{J+1}$, so that $a_J{}^+$ and $b_J{}^+$ refer to $N\bar{N}$ in the ${}^3J\pm1_J$ state, $a_J{}^-$ to the ${}^3J_J$ state, and $b_J{}^-$ to the ${}^1J_J$ state. The first two combinations of amplitudes $\bar{f}^t$ in Table I,

$$\bar{f}_{01,\frac{1}{2}\frac{1}{2}}{}^t\pm\bar{f}_{01,-\frac{1}{2}-\frac{1}{2}}{}^t=\sum_J(2J+1)\times(T_{01,\frac{1}{2}\frac{1}{2}}{}^J\pm T_{01,-\frac{1}{2}-\frac{1}{2}}{}^J)P_{J-1}{}^{(1,1)}\,, \tag{2.7}$$

are then easily expressed in terms of the b's, while the last two combinations in Table I,

$$\bar{f}_{01,\frac{1}{2}-\frac{1}{2}}{}^t\pm\bar{f}_{01,-\frac{1}{2}\frac{1}{2}}{}^t=\sum_J(2J+1)\times\{T_{01,\frac{1}{2}-\frac{1}{2}}{}^JP_{J-1}{}^{(2,0)}\pm T_{01,-\frac{1}{2}\frac{1}{2}}{}^JP_{J-1}{}^{(0,2)}\}\,, \tag{2.8}$$

can be expressed in terms of the a's. The results are listed in Table I.[15] Finally, by inserting the appropriate partial-wave expansions in Eq. (2.3) we obtain

$$\sum(2J+1)b_J{}^-P_{J-1}{}^{(1,1)}(\cos\theta_t)=-i\sum(J+\tfrac{1}{2})\times[a_J{}^-(P_{J-1}{}^{(2,0)}-P_{J-1}{}^{(0,2)})+a_J{}^+(P_{J-1}{}^{(2,0)}+P_{J-1}{}^{(0,2)})]\,. \tag{2.9}$$

This is the familiar form of the conspiracy relation, from which one can deduce ratios of the residues of the various conspiring trajectories.

III. CONSEQUENCES OF CONSPIRACY FOR THE PHOTOPRODUCTION CROSS SECTION

The differential cross section for photoproduction by unpolarized initial beams can be written

$$\frac{d\sigma}{d\Omega}=\frac{p_f}{16\pi^2sp_i}\sum_{abcd}|f_{cd,ab}{}^t|^2\,, \tag{3.1}$$

where Ω, p_i, and p_f represent the solid angle, initial momentum, and final momentum, all in the center-of-mass system. Using the relations between $f_{cd,ab}{}^t$ and $f_{-c-d,-a-b}{}^t$ provided by parity conservation to reduce the number of amplitudes, we can rewrite (3.1) as

$$\frac{d\sigma}{d\Omega}=\frac{p_f}{16\pi^2sp_i}(|f_{01,\frac{1}{2}\frac{1}{2}}{}^t+f_{01,-\frac{1}{2}-\frac{1}{2}}{}^t|^2+|f_{01,\frac{1}{2}\frac{1}{2}}{}^t-f_{01,-\frac{1}{2}-\frac{1}{2}}{}^t|^2+|f_{01,\frac{1}{2}-\frac{1}{2}}{}^t+f_{01,-\frac{1}{2}\frac{1}{2}}{}^t|^2+|f_{01,\frac{1}{2}-\frac{1}{2}}{}^t-f_{01,-\frac{1}{2}\frac{1}{2}}{}^t|^2)\,. \tag{3.2}$$

The amplitudes f^t are related to $\bar{f}^t$ by Eq. (2.1), and $\bar{f}^t$ to the kinematic singularity-free $\tilde{f}^t$ by (2.2).

Most of the t dependence of $d\sigma/d\Omega$ near $t=0$ should be determined by the half-angle factors $(\sin\frac{1}{2}\theta_t)^{|\lambda-\mu|}$ $(\cos\frac{1}{2}\theta_t)^{|\lambda+\mu|}$, the kinematic factors $\bar{K}(t)$, and the extra zeros which appear in certain $\tilde{f}^t$ in the absence of conspiracy. The remaining factors in $\tilde{f}^t$ are dynamical and are expected to vary only slowly at small t. [One would expect the pion exchange pole to provide an

[15] Note that as a result of the property
$$P_{J-1}{}^{(0,2)}(\cos\theta_t)=(-1)^{J-1}P_{J-1}{}^{(2,0)}(-\cos\theta_t)\xrightarrow[\cos\theta_t\to\infty]{}(\cos\theta_t)^{J-1},$$
the combination $(P_{J-1}{}^{(2,0)}+P_{J-1}{}^{(0,2)})$ dominates $(P_{J-1}{}^{(2,0)}-P_{J-1}{}^{(0,2)})$ in the Reggeized amplitude at large $\cos\theta_t$ by one power of $\cos\theta_t$, making it possible to speak of a "dominant parity" in the last two amplitudes.

TABLE III. The low-t behavior of each photoproduction helicity amplitude, with and without conspiracy, when the dynamical factors and $(t-4M^2)$ are approximated by constants c_i. The contribution of each helicity amplitude in this approximation to $d\sigma/d\Omega$ [the relation of $\cos\theta_t$ to t is given by $\cos^2\theta_t = t(2s+t-2M^2-\mu^2)^2/(t-\mu^2)^2(t-4M^2)$].

Amplitude	Behavior without conspiracy	Behavior with conspiracy	Contribution to $(16\pi^2 sp_i/p_f)d\sigma/d\Omega$ Without conspiracy	With conspiracy
$\bar{f}_{01,\frac12\frac12}{}^t+\bar{f}_{01,-\frac12-\frac12}{}^t$	$c_1(t-\mu^2)$	$c_1(t-\mu^2)$	$\frac{\lvert c_1\rvert^2}{4}(t-\mu^2)^2\sin^2\theta_t$	$\frac{\lvert c_1\rvert^2}{4}(t-\mu^2)^2\sin^2\theta_t$
$\bar{f}_{01,\frac12\frac12}{}^t-\bar{f}_{01,-\frac12-\frac12}{}^t$	$c_2t^{1/2}$	$c_2/t^{1/2}$	$\frac{\lvert c_2\rvert^2}{4}t\sin^2\theta_t$	$\frac{\lvert c_2\rvert^2}{4t}\sin^2\theta_t$
$\bar{f}_{01,\frac12-\frac12}{}^t+\bar{f}_{01,-\frac12\frac12}{}^t$	$c_3t^{1/2}(t-\mu^2)$	$c_3(t-\mu^2)/t^{1/2}$	$\frac14\lvert c_3\rvert^2t(t-\mu^2)^2[1+\cos^2\theta_t]$ $+\frac14\lvert c_4\rvert^2[1+\cos^2\theta_t]$	$\frac14\lvert c_3\rvert^2t^{-1}(t-\mu^2)^2[1+\cos^2\theta_t]$ $+\frac14\lvert c_4\rvert^2[1+\cos^2\theta_t]$
$\bar{f}_{01,\frac12-\frac12}{}^t-\bar{f}_{01,-\frac12\frac12}{}^t$	c_4	c_4	$-\mathrm{Re}c_3{}^*c_4t^{1/2}(t-\mu^2)\cos\theta_t$	$-t^{-1/2}\,\mathrm{Re}c_3{}^*c_4(t-\mu^2)\cos\theta$

TABLE IV. Sample values of kinematic factors for $k_\gamma=3$ BeV/c.

	Factor	$t=-0.02$ BeV^2	$t=-0.003$ BeV^2 (2.8° c.m.)	$t=t_{\min}$ (0° c.m.)
(i)	$-(t-\mu^2)^2\sin^2\theta_t$	0.72 BeV^4	0.11 BeV^4	0
(ii)	$t\sin^2\theta_t$	9.2 BeV^2	0.64 BeV^2	0
(ii)′	$\sin^2\theta_t/t$	2.3×10^4 BeV^{-2}	7.1×10^4 BeV^{-2}	0
(iii)	$-t(t-\mu^2)^2[1+\cos^2\theta_t]$	1.4×10^{-2} BeV^6	3.3×10^{-4} BeV^6	7.8×10^{-9} BeV^6
(iii)′	$-(1/t)(t-\mu^2)^2[1+\cos^2\theta_t]$	36.1 BeV^2	36.2 BeV^2	72 BeV^2
(iv)	$[1+\cos^2\theta_t]$	460	214	2
(v)	$\lvert t^{1/2}(t-\mu^2)\cos\theta_t\rvert$	1.2×10^{-1} BeV^3	1.8×10^{-2} BeV^3	6.6×10^{-5} BeV^3
(v)′	$\lvert(1/t^{1/2})(t-\mu^2)\cos\theta_t\rvert$	6 BeV	6 BeV	6 BeV

exception, but we show in Appendix A that the pion pole $(\sin\pi\alpha_\pi)^{-1}$ is cancelled in $\bar{f}^t$ by a factor α_π, to be restored by a kinematic pole from the half-angle factor. Thus, all $\bar{f}^t$ are smooth functions below the 2π threshold at $t=4m_\pi^2$.]

In order to study the difference in t dependence between the situations with and without conspiracy, then, we approximate the dynamical factors by constants at small t. With this approximation, the helicity amplitudes $\bar{f}^t$ take the forms listed in Table III. Relating $\bar{f}^t$ to f^t by Eq. (2.1) and squaring, we obtain the contributions to $d\sigma/d\Omega$ (3.2) given in the right-hand columns of Table III. In Table IV, the variation of these contributions at the sample energy $k_\gamma=3$ BeV/c is indicated by a numerical evaluation at $t=-0.02$ BeV^3 (i.e., a point outside the dip region), $t=-0.003$ BeV^2 (i.e., at center-of-mass angle $\theta_s=2.8°$, which is near the most forward point measured at present), and $t=t_{\min}$ (where $\theta_s=0°$).

The numerical evaluation at $k_\gamma=3$ BeV/c works out as follows. In the absence of conspiracy, all kinematic factors in Table IV (i, ii, iii, iv, and v) tend to decrease from $t=-0.02$ to $t=-0.003$, which corresponds to the most forward region measured by Buschhorn *et al.*[10] The decrease by a factor of ~100 between $t=-0.003$ BeV^2 and $t_{\min}\approx0$ is even more striking, and not likely to be counteracted by any dynamical effect. Thus, in the absence of conspiracy, there should be a sharp drop in $d\sigma/d\Omega$ in the forward direction (θ_s less than 2.5° in the center of mass). This drop is already well known for pion exchange, and our derivation shows it occurs independently of the particle exchanged.

Kinematic factors (ii)′, (iii)′, and (v)′, which replace (ii), (iii), and (v) in the presence of conspiracy, tend to show the forward rise indicated by experiment as $\theta_{\text{c.m.}}$ decreases to 2.5°. At still smaller angles, the difference between the cases with and without conspiracy becomes more dramatic; (v)′ is constant, (iii)′ remains essentially constant from $\theta_{\text{c.m.}}=2.5$ to $\theta_{\text{c.m.}}=0.5°$ before doubling at $\theta_{\text{c.m.}}=0°$, and (ii)′ remains essentially constant from $\theta_{\text{c.m.}}=2.5°$ to $\theta_{\text{c.m.}}=0.5°$ before plunging to zero at $\theta_{\text{c.m.}}=0°$. Thus the deep forward dip predicted by standard techniques for Reggeizing individual exchanges should be absent if conspiring trajectories dominate the cross section.[16]

The same qualitative features apply at energies below 3 BeV. While the dip which distinguishes the no-conspiracy from the conspiracy case becomes less striking at low energies, the depth being of order 10 $(E_\gamma/\text{BeV})^2$, the effect persists below 1 BeV.[17] Thus, for example, the measurements at $E_\gamma=800$ MeV by Beneventano *et al.*,[10] who find $d\sigma/d\Omega$ rising as $\theta_{\text{c.m.}}$ is

[16] Of course, conspiracy does permit *some* decrease of the forward cross section if the Regge α has a special value such that the c_2 term dominates c_3 and c_4, or c_3 and c_4 have a strong destructive interference.

[17] Note that no high-energy approximation has been made in Table III.

decreased by steps of 2° from 10° to 0°, are relevant and strongly favor conspiracy. The measurements down to 5° by Ecklund and Walker at somewhat higher energies, and down to 2.5° by Buschhorn *et al.* at 1.2 to 2.9 BeV, support the same conclusion. If this pattern is corroborated by extension of the higher-energy measurements to 0°, the existence of conspiracy will be firmly established.

Note added in proof. As noted above, our analysis is not limited to high energies, and the dip distinguishing between the no-conspiracy and conspiracy cases remains visible down into the resonance region. It is interesting that the Feynman diagrams for individual s-channel resonances do *not* have dips at $t=0$. This provides another indication that conspiracy among amplitudes takes place.

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APPENDIX A: KINEMATIC SINGULARITIES AND EXTERNAL PHOTONS

A standard method[11] of determining the kinematical factor $\bar{K}(t)$ is to examine the crossing matrix between t-channel and s-channel helicity amplitudes. The amplitudes $\bar{f}_{cd,ab}{}^t$ defined by

$$\bar{f}_{cd,ab}{}^t=\frac{f_{cd,ab}{}^t}{(\sin\frac{1}{2}\theta_t)^{|c-d-a+b|}(\cos\frac{1}{2}\theta_t)^{|c-d+a-b|}} \qquad \text{(A1)}$$

are assumed to have no kinematic singularities in s; the same assumption is made about the singularities in t of $\bar{f}^s$. Then study of the crossing matrix $\bar{X}$ in $\bar{f}_i{}^t=\bar{X}_{ij}\bar{f}_j{}^s$ (where i and j represent sets of helicity indices) allows a determination of the kinematic singularities of each amplitude.

This method has been employed by Wang[11,18] to determine singularities of the parity-conserving helicity amplitudes[12] $\bar{f}_{cd,ab}{}^t\pm\bar{f}_{cd,-a-b}{}^t$ for the case of vector-meson production. These agree with conventional arguments about threshold and pseudothreshold behavior.[5] The $\bar{K}$ listed in Table I were obtained by calculating with this method at $m_V\neq0$, and then taking $\lim_{m_V\to0}\bar{K}$. Note that two of the $\bar{K}$ in Table I contain a factor $(t-\mu^2)$ and the other two do not.

Study of the explicit crossing matrix for photoproduction evaluated directly at $m_\gamma=0$, on the other hand, gives a factor $(t-\mu^2)$ in the $\bar{K}$ for *all four* helicity amplitudes. An analogous discrepancy has been found by Horn[19] in the reaction $\gamma\gamma\to\pi\pi$. Thus the determination of kinematic singularities from crossing matrix considerations must be reexamined when one or more of the external particles has zero mass.

[18] L. L. Wang, Phys. Rev. **153**, 1664 (1967).

In order to decide which prescription for $\bar{K}$ is more suitable—the limit as $m_V\to0$ of $\bar{K}$ calculated from the crossing matrix for $m_V\neq0$, or $\bar{K}$ calculated directly from the crossing matrix for $m_V=0$—let us turn to the Reggeization of pion exchange. We Reggeize parity-conserving helicity amplitudes in the manner developed by Wang.[18] This formalism implies that the contribution of a single Regge trajectory to a given helicity amplitude is, to highest order in s,

$$f_{cd;ab}{}^t\underset{s\to\infty}{\longrightarrow}(\sin\tfrac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\tfrac{1}{2}\theta_t)^{|\lambda+\mu|}\times\frac{\bar{K}(t)\gamma(t)s^{\alpha-M}(1\pm e^{-i\pi\alpha})}{\sin\pi\alpha}, \qquad \text{(A2)}$$

where $M=\max(|\lambda|,|\mu|)$ and $\gamma(t)$ is the dynamical residue function.

The dynamical pion pole contributes only to the particular parity-conserving amplitude

$$\bar{f}_{01;\frac{1}{2}\frac{1}{2}}{}^t-\bar{f}_{01;-\frac{1}{2}-\frac{1}{2}}{}^t=\sum(2J+1)b_J{}^-(t)P_{J-1}{}^{(1,1)}(\cos\theta_t)\,.$$

Because Reggeization of $P_{J-1}{}^{(1,1)}(x)$ leads to a term proportional to $\alpha s^{\alpha-1}$, the contribution of pion trajectory exchange to amplitudes $f_{01;\frac{1}{2}\frac{1}{2}}{}^t$ and $f_{01;-\frac{1}{2}-\frac{1}{2}}{}^t$ assumes the form

$$f_{01;\,\lambda_1\lambda_1}{}^t\to(\sin\tfrac{1}{2}\theta_t)(\cos\tfrac{1}{2}\theta_t)\times\left(\frac{t-4M^2}{t}\right)^{1/2}\frac{\gamma(t)\alpha s^{\alpha-1}(1+e^{-i\pi\alpha})}{\sin\pi\alpha}, \qquad \text{(A3)}$$

where we have inserted $\bar{K}(t)$ from Table I. As first noticed by Zweig[20] and Dombey,[21] the pion pole in this expression emerges in a curious way. The dynamical contribution is proportional to $\alpha_\pi/\sin\pi\alpha_\pi$, which contains no pole at $\alpha_\pi=0$. However, the kinematic factor

$$2\sin\tfrac{1}{2}\theta_t\cos\tfrac{1}{2}\theta_t=\sin\theta_t=(1-\cos^2\theta_t)^{1/2}$$

introduces a pole at $t=\mu^2$ because

$$\cos^2\theta_t=t[2s+t-2M^2-\mu^2]^2(t-4M^2)^{-1}(t-\mu^2)^{-2}\,.$$

Thus the pion pole arises kinematically when the $\bar{K}$ of Table I is used.

The alternative $\bar{K}$, obtained by working directly with the crossing matrix at $m_\gamma=0$, has an additional factor $t-\mu^2$. If this $\bar{K}$ were used in the above formalism, it would cancel the $t=\mu^2$ pole from the half-angle factors, and the resulting amplitude would have no pion pole. This would contradict gauge-invariant perturbation theory, where f^t does have a pion pole, and the experimental observation that such a pole is necessary to fit

[19] D. Horn, California Institute of Technology Report No. CALT-68-131/Internal Report 34, 1967 (unpublished).

[20] G. Zweig, Nuovo Cimento **32**, 689 (1964).

[21] N. Dombey, Nuovo Cimento **32**, 1696 (1964).

charged photoproduction data. Hence, we feel that the $\bar{K}$ listed in Table I are to be preferred.[22,23]

[*Note added in proof.* It has been pointed out to us by Dr. Frank Henyey that the above discussion is incomplete. While our choice of kinematic factors allows pion exchange to contribute the expected pole at $t=\mu^2$, it also allows exchange of any *other* particle with unnatural parity to contribute a pole at $t=\mu^2$ in A_2 or A_3 (regardless of the mass of the exchange). That this does not happen can easily be seen in gauge-invariant perturbation theory, where only renormalized pion exchange contributes a pole at $t=\mu^2$.]

Thus all terms in A_3, and all $P=(-1)^{J+1}$ terms in A_2 *except* pion exchange, contain an extra factor of $(t-\mu^2)$. Whether this factor should be regarded as "kinematic" or "dynamical" we do not know. Fortunately it has little effect on the analysis of the forward dip, as pointed out in Ref. 23.

APPENDIX B: DERIVATION OF CONSPIRACY RELATIONS FROM CROSSING

It is natural to ask whether the procedure followed in this paper for photoproduction can be used to study conspiracy in reactions with arbitrarily high spin. Looking back over the procedure of Sec. II, one sees that we worked directly with the helicity amplitudes and their crossing properties in setting up Table I, and then switched over to the relations between helicity amplitudes and the invariant amplitudes A_i. Prescriptions for the helicity amplitudes are available in the general spin case, from the work of Wang,[12] but prescriptions for invariant amplitudes are not. It is therefore of interest that there exists an alternative derivation of the nucleon-nucleon conspiracy relation, and of the photoproduction conspiracy relation in the high-energy limit, which employs only helicity amplitudes and their crossing properties.

In nucleon-nucleon scattering, $t=0$ corresponds to $\theta=0°$ in the s-channel center-of-mass system. At this point, many s-channel helicity amplitudes vanish by angular-momentum conservation. When this information is used as input to the crossing relations from f^s to f^t, one obtains precisely the usual NN conspiracy relation for the t-channel amplitudes.

The analogous argument for photoproduction is more complicated. In the first place, because of the unequal masses, $\theta=0°$ in the s-channel corresponds to $t=0$ only in the limit $s\to\infty$. Secondly, no conspiracy relations are obtained from the crossing equation $f^t=Xf^s$ in this limit, because along the $\theta_s=0°$ curve in the s, t plane, each f^s which is nonzero by angular-momentum conservation crosses to only one nonzero f^t.[24]

The conspiracy relation (2.3) for photoproduction, however, involves the amplitudes $\bar{f}^t$ rather than f^t. This suggests that we start with the equation $\bar{f}^t=\bar{X}\bar{f}^s$. It has been proved by Wang that the crossing matrix $\bar{X}$ is nonsingular along the curve $\theta_s=0°$.[25] By studying the crossing in the limit $\theta_s=0°$, $s\to\infty$, we find[26]

$$\bar{f}_{01,\frac{1}{2}\frac{1}{2}}{}^t-\bar{f}_{01,-\frac{1}{2}-\frac{1}{2}}{}^t\to -2i\bar{f}_{0\frac{1}{2},-1-\frac{1}{2}}{}^s\,, \tag{B1}$$

$$\bar{f}_{01,\frac{1}{2}-\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}\frac{1}{2}}{}^t\to 2\bar{f}_{0\frac{1}{2},-1-\frac{1}{2}}{}^s\,, \tag{B2}$$

which imply the relation (2.3):

$$\bar{f}_{01,\frac{1}{2}\frac{1}{2}}{}^t-\bar{f}_{01,-\frac{1}{2}-\frac{1}{2}}{}^t=-i(\bar{f}_{01,\frac{1}{2}-\frac{1}{2}}{}^t+\bar{f}_{01,-\frac{1}{2}\frac{1}{2}}{}^t) \tag{B3}$$

in the high-energy limit.

Note added in proof. Recently, Cohen-Tannoudji *et al.*[27] have given a general method for deriving conspiracy relations from the crossing matrix. Applying their method, one can derive Eq. (II3) at all s. The work of Cohen-Tannoudji *et al.* confirms that the information on conspiracy is contained in the crossing matrix, and thus can be found without direct recourse to the invariant amplitudes or to $O(4)$ symmetry.

[22] In other words, if one uses the other $\bar{K}$, a kinematic pole must be inserted at $t=\mu^2$ in the resulting $\bar{f}$, in order to obtain the pion pole observed in nature.

[23] Notice that in the extremely low t region where, according to Sec. III, the main distinction between conspiracy and evasion occurs, factors of $(t-\mu^2)$ have little effect. Thus our choice of threshold behavior does not affect conclusions about the presence of conspiracy.

[24] Those f^t which contain factors $\cos\theta_t/2$ vanish because $\theta_t=180°$ along the curve $\theta_s=0°$.

[25] The explicit form of $\bar{X}$ is given by Wang [Ref. 12, Eq. (III4)], and the proof that it is not singular along the curve $\theta_s=0°$ is given in her Appendix B.

[26] Further terms of order $s^{-\frac{1}{2}}\bar{f}^s$ on the right-hand side have been dropped in the limit $s\to\infty$.

[27] G. Cohen-Tannoudji, A. Morel, and H. Navelet, Kinematic Singularities, Crossing Matrix, and Kinematical Constraints for Two-Body Helicity Amplitudes, Saclay Report, 1967 (unpublished).